

Amendments to Claims:

This listing of claims will replace all prior versions, and listings, of claims in the application:

Listing of Claims:

Claim 1 (currently amended):

A computer implemented ~~method~~ system for developing an optimized workforce schedule for a plurality of agents, each with a combination of defined skills and belonging to a skill group with agents having the same skills, to serve one or more contact types such as telephone calls, email, and other interaction media, a plurality of ~~contact types, part time/full time agent work groups with~~ tour types with defined ~~and shift~~ scheduling ~~rules~~ rules, located at one or more contact centers each with its own operating hours and time zones, comprising the steps of:

- (a) acquiring ~~net staffing level~~ agent requirements, b_{ht}^r for each contact type, and for each period and day to be scheduled by a computer ~~any means~~;
- (b) acquiring tour, shift, days-off, and break scheduling rules, agent ~~skills groups~~, ~~part-time/full-time agent work~~ skill groups, agent availability, and objective criterion to be optimized and its parameters by a computer ~~any means~~;
- (c) formulating the constraints and objective function of a Mixed Integer Programming (MILP) model ~~to describe with the tour types, shift, and days-off~~ and associated scheduling rules, rules including consistent and non-consistent daily start time requirements, a plurality of relief and lunch breaks each with a duration of one or more planning periods and a break window during which the break must be started and completed, ~~staffing level~~ agent requirements for a plurality of contact types and for each period to be scheduled, a plurality of agent, ~~part-time/full-time agent~~ skills and skill groups, agent availability, and agent costs by a computer;
wherein formulating the constraints and objective function of the MILP model comprises:

$$\begin{aligned}
& \text{Minimize } \sum_{j \in J} \sum_{i \in QI_k} \sum_{k \in OK_j} C_{kl}^j Q_{kl}^j \\
& + \sum_{j \in J} \sum_{k \in OK_j} \sum_{n \in Fk} \sum_h \sum_{i \in QI_k} c_{knh}^j QX_{knh}^j \\
& + \sum_{r \in R} \sum_{t \in T_h} \sum_h P_{ht}^r S_{ht}^r \quad (c1)
\end{aligned}$$

Subject to

$$\begin{aligned}
& \sum_{j \in Mr} e^{jr} G_{ht}^{jr} + S_{ht}^r - O_{ht}^r = b_{ht}^r \\
& , r \in R, t \in T_h, h = 1, \dots, 7, \quad (c2)
\end{aligned}$$

$$\begin{aligned}
& f_{ht}^j(QX, QU, QW, QV) - \sum_{r \in N_j} G_{ht}^{jr} = 0 \\
& , j \in J, t \in T_h, h = 1, \dots, 7, \quad (c3)
\end{aligned}$$

$$\begin{aligned}
& QX_{knh}^j = \sum_{t \in QB1knh} QU_{kniht}^j \\
& , j \in J, n \in Fk, i \in QI_k, k \in OK_j, h = 1, \dots, 7, \quad (c4)
\end{aligned}$$

$$\begin{aligned}
& QX_{knh}^j = \sum_{t \in QB2knh} QW_{kniht}^j \\
& , j \in J, n \in Fk, i \in QI_k, k \in OK_j, h = 1, \dots, 7, \quad (c5)
\end{aligned}$$

$$\begin{aligned}
& QX_{knh}^j = \sum_{t \in QB3knh} QV_{kniht}^j \\
& , j \in J, n \in Fk, i \in QI_k, k \in OK_j, h = 1, \dots, 7, \quad (c6)
\end{aligned}$$

$$\begin{aligned}
& \sum_{i \in QI_k} A_{kjh} Q_{kl}^j = \sum_{n \in Fk} \sum_{i \in QI_k} QX_{knh}^j \\
& , j \in J, k \in OK_j, h = 1, \dots, 7, \quad (c7)
\end{aligned}$$

$$\begin{aligned}
& \sum_{i \in QI_k} Q_{kl}^j < QD_k^{j \max} \\
& , j \in J, k \in OK_j, \quad (c8)
\end{aligned}$$

$$\begin{aligned}
& QX_{knh}^j, Q_{kl}^j, QU_{kniht}^j, QW_{kniht}^j, \text{ and } QV_{kniht}^j \geq 0 \text{ and integer for all } j, k, n, i, h, t, \\
& \text{and } G_{ht}^{jr}, S_{ht}^r, \text{ and } O_{ht}^r \geq 0 \text{ for all } j, r, h, \text{ and } t, \quad (c9)
\end{aligned}$$

where, in constraint (c3),

$$\begin{aligned}
f_{ht}^j(QX, QU, QW, QV) = & \sum_{k \in QK_j} \sum_{n \in F_k} \sum_{i \in QI_k} a_{kniht} QX_{kniht}^j \\
& - \sum_{k \in QK_j} \sum_{n \in F_k} \sum_{i \in QT1knh} QU_{kniht}^j \\
& - \sum_{k \in QK_j} \sum_{n \in F_k} \sum_{i \in QT2knh} (QW_{kniht}^j + QW_{kniht-1}^j) \\
& - \sum_{k \in QK_j} \sum_{n \in F_k} \sum_{i \in QT3knh} QV_{kniht}^j
\end{aligned}$$

, $j \in J, t \in T_h, h = 1, \dots, 7, \quad (c10)$

where

F_k is the set of shift lengths specified for tour k ;

J is the set of all skill groups;

R is the set of all contact groups;

T_h is the set of all planning periods in day h ;

QI_k is the set of daily start times for tour k ;

QK_j is the set of all possible tours including both requiring and not requiring consistent daily start times for scheduling agents in skill group j . When a tour requires consistent daily start times, a pseudo tour is defined for each start time of the tour with the same tour, shift, break, and work and non-work rules, and included in QI_k and QK_j ;

QL_k is the set of all allowed work day patterns for the agents assigned to tour k . Only the work patterns satisfying the work and non-work day rules specified for tour k are included in QL_k ;

$QB1_{kniht}$ is the set of planning periods on day h during which an agent assigned to tour k and shift length n with a daily start time of i may start his/her first relief break and complete within the time window specified

for this tour group, shift length, start time, and day;

QB2_{knih} is the set of planning periods on day h during which an agent assigned to tour k and shift length n with a daily start time of i may start his/her lunch break and complete within the time window specified for this tour group, shift length, start time, and day;

QB3_{knih} is the set of planning periods on day h during which an agent assigned to tour k and shift length n with a daily start time of i may start his/her second relief break and complete within the time window specified for this tour group, shift length, start time, and day;

QT1_{knht} is the set of daily start times for shift length n for tour k for which period t on day h is a first relief break start period;

QT2_{knht} is the set of daily start times for shift length n for tour k for which period t on day h is a lunch break start period;

QT3_{knht} is the set of daily start times for shift length n for tour k for which period t on day h is a second relief break start period;

M^r is the set of skill groups that can serve contact type r;

N_j is the set of contact types that skill group j is qualified to provide service;

a_{knih} is equal to one when period t on day h is in the shift span (that is, a work or a break period) of agents assigned to tour k who have a daily start time of i and shift length n on day h, and zero otherwise;

A_{klh} is equal to one when day h is a work day for agents assigned to tour k and work pattern l, and zero otherwise;

C_{kl}^j is the weekly cost of assigning an agent in skill group j to tour k with work pattern of l;

c_{knhi}^j is the daily wage paid in addition to the weekly cost of C_{kl}^j to agents in skill group j assigned to tour type k with a daily shift length of n and start time of i on day h;

b_{ht}^r is the number of agents with the highest skill proficiency to serve contact type r ∈ R required in period t on day h;

P_{ht}^r is the per-unit penalty cost for allocating fewer than the number of agents with skill to serve contact type r ∈ R required b_{ht}^r, in period t on day h;

e^{jr} is the relative efficiency of an agent from skill group j in serving contact type r with respect to an agent with the highest skill proficiency (100% efficiency level) for contact type r , $e^{jr} \in [0, 1]$;

$QD_k^{j\max}$ is the maximum number of agents available in skill group j to assign to tour k ;

where decision variables whose values are determined by a solution to the MILP model are defined as:

shift variables:

QX_{knh}^j is the number of agents in skill group j assigned to tour k and shift length n with a daily starting time of i on day h ;

break variables:

QU_{knh}^j is the number of agents in skill group j assigned to tour k and shift length n with a daily start time of i and starting their first relief breaks in period t on day h ;

QW_{knh}^j is the number of agents in skill group j assigned to tour k and shift length n with a daily start time of i and starting their lunch breaks in period t on day h . When a tour has more than one lunch break to be scheduled during a shift, then a set of break variables and constraints are defined for each lunch break type in the same manner with QW_{knh}^j variables and constraints (c5), and included in constraint (c3). A lunch break variable may be two or more periods long. A lunch break variable will appear in all periods on the left hand side of (c3) with a negative sign during which the agents assigned to the associate break start time will be on break;

QV_{knh}^j is the number of agents in skill group j assigned to tour k and shift length n with a daily start time of i and starting their second relief breaks in period t on day h . When a tour has more than two relief breaks to be

scheduled during a shift, then a set of break variables and constraints are defined for each break type in the same manner with QU_{kniht}^j and QV_{kniht}^j variables and constraints (c4) and (c6), and included in constraint (c3)

A relief break variable may be one or more periods long. A relief break variable will appear in all periods on the left hand side of (c3) with a negative sign during which the agents assigned to that break start time will be on break;

work pattern variables:

Q_{kl}^j is the number of agents in skill group j assigned to tour k with a work pattern of l;

allocation variables:

G_{ht}^{jr} is the number of agents from skill group j scheduled and not on a break in planning period t on day h, and allocated to contact type r;

shortage variables:

S_{ht}^r is the total agent shortages on the left hand side of constraint (c2) in meeting the required number of agents, b_{ht}^r , with skill to serve contact type $r \in R$ in planning period t on day h;

excess variables:

O_{ht}^r is the total overstaffing on the left hand side of constraint (c2) in excess of the required number of agents, b_{ht}^r , with skill to serve contact type $r \in R$ in planning period t on day h;

variable sets

$QX = \{ QX_{knh}^j: j \in J, n \in F_k, k \in OK_j, i \in I_k, h = 1, \dots, 7 \}$ is the set of shift variables

QX_{knh}^j

QU, QW, and QV are defined similar to the set QX to include, respectively, the first relief break, lunch, and second relief break variables (e.g. the set QU includes QU_{knh}^j , QW includes QW_{knh}^j) for skill groups in J, $n \in F_k, k \in OK_j, i \in I_k, h = 1, \dots, 7$, and $t \in QB1_{knh}$ for QU, $t \in QB2_{knh}$ for QW, and $t \in QB3_{knh}$ for QV. To facilitate the presentation of function (c10), planning period index (t-1) is used. If (t-1) is in one day and planning period t in the next day, planning period and day indexes are adjusted accordingly. For example, if a planning period is 15 minutes long and period 96 is starting period for a lunch break variable on day h, then the second planning period for this variable is not period 97, which is in the next day, but period 1 of day (h+1). Likewise if (h+1) is beyond the end of the scheduling period, schedules “wrap around” and agent availabilities appear in the first of the scheduling period. If some tours have four or more break types to be scheduled, one set for each break type is defined in the same manner with QU, QW, and QV. The function $f_{ht}^j(QX, QU, QW, QV)$ includes the decision variables for skill group j in QX, and the sets of break variables (e.g. QU, QW, QV) only,

and

$G = \{ G_{ht}^r: j \in J, r \in R, t \in T_h, h = 1, \dots, 7 \}$ is a set of variables G_{ht}^r

- (d) solving obtaining the Linear Programming (LP) LP relaxation of the MILP model formulated in the B&C Branch and Cut (B&C) algorithm by relaxing all integrality constraints on the decision variables, solving the LP relaxation, and stopping the B&C algorithm with an optimal solution to the MILP model if when the optimal solution of the LP relaxation satisfies all integrality constraints, by a computer –an optimal solution to the MILP model is found;

- (e) calling the Rounding Algorithm (RA) RA-algorithm consisting of the following steps by a computer if when the a solution to a node the LP relaxation of the MILP model violating some integrality constraints is found by the B&C algorithm:
- i. obtaining the values of the decision variables in the optimal solution found for the LP relaxation of the MILP model;
 - ii. rounding the fractional values of decision variables in QX, QU, QW, QV, and G down, and weekly tour variables QX, and work pattern variables Q_{kl}^j down when their fractional part is less than or equal to 0.50, and up if greater than 0.50, provided the agent availability constraints on the maximum number of agents available are not violated;
 - iii. scheduling additional shifts by increasing the values of decision variable in QX if additional shifts are needed to satisfy the number of work days required by tour scheduling rules for each agent, and to have a shift scheduled for every agent who will be working on a given day based of on the work patterns Q_{kl}^j scheduled;
 - iv. scheduling additional daily breaks when the number of breaks of each type is not sufficient to satisfy break scheduling rules for a tour for each day, daily shift length and start time, and unscheduling breaks when there are more breaks are scheduled than required by a tour for a day, shift length and start time, due to rounding;
 - v. computing the left side of constraint (c2) using the values determined in steps (1-iv) for the decision variables, subtracting the right side from the left side of constraint (c2, and determining agent shortages S_{ht}^r , when the difference is negative, and excesses O_{ht}^r , when the difference is positive, for each contact type and planning period;
 - vi. computing scheduled agent availability for all skill groups and planning periods in (c10), comparing them with the agent allocations to different contact types by the rounded values of allocation variables G, and adjusting the values of the associated allocation, shortage, $S_{ht,s}^r$ and

excess, O'_{ht} , variables to make agent allocations equal to scheduled agent availability in (c10) and satisfy (c2);

- vii. checking the solution constructed in steps (i) through (vi) to determine if all agent requirements are met in every planning period and , when all requirements are met, eliminating all redundant agent tours tour schedules that do not create agent shortages in any period when removed by lowering the values of related shift, break, and work pattern variables and stopping with the integer feasible solution found;
- viii. continuing to step (xii) with an integer feasible solution with agent shortages when agent requirements for some contact types are not met in all periods and all available agents in constraint (c8) who can serve these contact types are scheduled;
- ix. continuing to step (x) when there are agents available to schedule when left hand side of (c8) is less than its right hand side for one or more skill groups and there shortages in some periods for some contact types served by these skill groups;
- x. finding all periods in which some contact types have shortages, finding an agent with skills needed, from a skill group for which the left side of (c8) is less than its right side, together with a complete tour schedule with work and non-work days, daily shift start times and shift lengths, daily break times to reduce the agent shortages for one or more contact types, and adding them to the solution by increasing the values of the corresponding decision variables by one to include the newly added agent tour schedule, and continuing to step (xii) when all agent requirements are met;
- xi. repeating steps (viii), (ix), and (x) until all agent requirements are met, or agent requirements for some contact types are not met in all periods and all available agents in constraint (c8) who can serve these contact types are scheduled;
- xii. examining the tours scheduled in the integer feasible solution found and eliminating redundant tours that do not create new agent shortages in

any period when removed by lowering the values of related shift, break,
and work pattern variables;

- (f) applying the RA algorithm to the solution found for the LP relaxation of the MILP model (current node) by the B&C algorithm when it violates one or more integrality conditions ~~and passing it to the B&C algorithm if and, when~~ a solution better than the best integer solution known is found by the RA algorithm, passing it to the B&C algorithm by a computer;

Claim 2 (original):

The method of claim 1 further comprising of repeating steps (e) and (f) for every node the B&C algorithm solves by the computer and finds a solution whose values for decision variables ~~violating~~ violate only some one or more integrality constraints, until a terminal solution ~~to the MILP model~~ that is either an optimal integer solution or the best integer solution for the MILP model is reached;

Claim 3 (original):

The method of claim 1 further comprising of processing the terminal solution found by the computer to assign daily shifts with start times and shift lengths to work patterns, ~~and days off scheduled to tours,~~ and daily breaks to specific shifts with start times and shift lengths to develop detailed weekly agent schedules;

Claim 4 (original):

The method of claim 1 further comprising of assigning agents to detailed weekly agent schedules ~~tours~~ by the computer;

Claim 5 (original):

The method of claim 1, in which a terminal solution to the MILP model formulated is found when the objective function value for an integer feasible solution differs no more than a pre-specified percentage from the ~~best lower bound found in the B&C algorithm for the MILP model~~ lowest objective function value found for the LP relaxations of all nodes created in the B&C algorithm to which the RA algorithm has not been applied when the integer feasible solution is found;

Claim 6 (original):

The method of claim 1, in which a terminal solution to the MILP model formulated is found when an integer feasible solution is found and a pre-specified period of time is passed in searching for a better integer feasible solution by the B&C algorithm;

Claim 7 (original):

The method of claim 1, in which a terminal solution to the MILP model formulated is found when an integer feasible solution is found and a pre-specified number of nodes are solved in the B&C algorithm and evaluated using the RA algorithm;

Claim 8 (cancelled)

Claim 9 (withdrawn):

~~A plurality of MILP models for optimal workforce scheduling in a contact center environment involving a plurality of agent skill groups who can serve customers using a plurality of contact types with the definitions of decision variables, formulations of objective functions, agent requirements constraints for each period and contact types to be scheduled, contact type assignment for agents from each agent skill group in each period to be scheduled, constraints to ensure tour, daily shift start time, daily shift length, days-off, work pattern, and a plurality of breaks and associated scheduling rules, and~~

~~constraints to adhere to the number of agents available in a plurality of part-time/full-time agent work groups.~~

Claim 10 (withdrawn):

~~The method of claim 9, in which the objective function, agent requirement constraint for each period and each contact type to be scheduled, and contact type assignment for agents from each agent skill group in each period to be scheduled are mathematically formulated as:~~

$$\begin{aligned} \text{Minimize } & \sum_{i \in I_k} \sum_{k \in K_j} C_{ki}^j X_{ki}^j + \sum_{i \in Q_L k} \sum_{k \in Q_K j} C_{kl}^j Q_{kl}^j \\ & + \sum_{k \in Q_K j} \sum_{n \in F_k} \sum_h \sum_{i \in Q_L k} C_{kni}^j Q X_{kni}^j \end{aligned}$$

Subject to

$$\sum_{j \in M_f} e^{jf} G_{ht}^{jf} \geq b_{ht}^f, \quad f \in R, t \in T_h, h = 1, \dots, 7,$$

$$\begin{aligned} f_{ht}^j(X, Y, Z, U, W, V, QX, Q, QU, QW, QV) \\ + \sum_{f \in N_j} G_{ht}^{jf} = 0, \end{aligned}$$

$$j \in J, t \in T_h, h = 1, \dots, 7,$$

where

$$\begin{aligned} f_{ht}^j(X, Y, Z, U, W, V, QX, Q, QU, QW, QV) = & \sum_{k \in K_j} \sum_{i \in I_k} a_{kith} X_{ki}^j \\ & - \sum_{k \in K_j} \sum_{i \in I_k} a_{kith} (Y_{kih}^j + Y_{ki(h-1)}^j) - \sum_{k \in K_j} \sum_{i \in I_k} \sum_{m=f_h(h-1)} a_{kith} Z_{kimh}^j \\ & - \sum_{k \in K_j} \sum_{i \in T1kht} U_{kith}^j - \sum_{k \in K_j} \sum_{i \in T2kht} (W_{kith}^j + W_{kih(t-1)}^j) \end{aligned}$$

$$\begin{aligned}
& - \sum_{k \in K_j} \sum_{i \in T3kht} V_{k iht}^j \\
& + \sum_{k \in QK_j} \sum_{n \in Fk} \sum_{i \in Q1k} a_{k n i h t} QX_{k n h i}^j \\
& - \sum_{k \in QK_j} \sum_{n \in Fk} \sum_{i \in Q1k} QU_{k n i h t}^j \\
& - \sum_{k \in QK_j} \sum_{n \in Fk} \sum_{i \in Q2k} (QW_{k n i h t}^j + QW_{k n i h (t-1)}^j) \\
& - \sum_{k \in QK_j} \sum_{n \in Fk} \sum_{i \in Q3k} QV_{k n i h t}^j
\end{aligned}$$

$$, j \in J, t \in T_h, h = 1, \dots, 7,$$

where $X_{k i}^j$ and $QX_{k n h i}^j$ are the agents from agent skill group j assigned to tour type k , respectively, requiring and not requiring consistent daily start times and shift length with a daily start time of i , $Q_{k i}^j$ is the number of agents from group j assigned to work pattern i of tour k , $Y_{k i h}^j$ is the number of agents from agent skill group j assigned to tour type k with a daily start time of i and starting their minimum required consecutive days off on day h , $Z_{k i m h}^j$ is the number of agents from agent skill group j assigned to tour type k with a daily start time of i with consecutive days off starting on day m and starting their additional days off (not necessarily consecutive) on day h , r_k is the number of additional days off an agent assigned to tour k receives in addition to consecutive days off, $U_{k i h t}^j$, $W_{k i h t}^j$ and $V_{k i h t}^j$ are the numbers of agents from agent skill group j assigned to tour type k , $k \in K_j$, with a daily start time of i and starting their relief or lunch break in period t on day h , $B1_{k i h}^j$, $B2_{k i h}^j$ and $B3_{k i h}^j$ are the break start times (break window) on day h for agents from agent skill group j assigned to tour k requiring consistent daily start times and shift lengths with a daily start time of i , $QU_{k i h t}^j$, $QW_{k i h t}^j$ and $QV_{k i h t}^j$ are the numbers of agents from agent skill group j assigned to tour type k , $k \in QK_j$, with a daily start time of i and starting their relief or lunch break in period t on day h , $QB1_{k i h}^j$, $QB2_{k i h}^j$ and $QB3_{k i h}^j$ are the break start times (break window) on day h for agents from agent skill group j assigned to tour k requiring consistent daily start times and shift lengths with a daily start time of i , $G_{h t}^r$ is the number agents from agent skill group j assigned to serve customers using type r contact type in period t on day h , $b_{h t}^r$ is the number of agents needed to serve customers using contact type r in period t on day h , M_r is the agent skill

~~groups that can serve customers using contact type r, R is the set of all contact types available to customers.~~

Claim 11 (withdrawn):

~~The method of claim 9, in which the break and days off scheduling requirements are formulated into a set of constraints requiring that sufficient number of daily breaks, and pre-specified minimum number of consecutive, and additional non-consecutive days off are scheduled for agents from each agent skill group assigned to tours requiring consistent daily shift start times as:~~

$$\begin{aligned}
 & \text{---} X_{ki}^j - Y_{kih}^j - Y_{ki(h-1)}^j - \sum_{m \neq h, (h-1)} Z_{kimh}^j - \sum_{t \in B1kih} U_{kiht}^j \text{---}, icI_k, k \in K_j, h = 1, \dots, 7, \\
 & \text{---} X_{ki}^j - Y_{kih}^j - Y_{ki(h-1)}^j - \sum_{m \neq h, (h-1)} Z_{kimh}^j - \sum_{t \in B2kih} W_{kiht}^j \text{---}, icI_k, k \in K_j, h = 1, \dots, 7, \\
 & \text{---} X_{ki}^j - Y_{kih}^j - Y_{ki(h-1)}^j - \sum_{m \neq h, (h-1)} Z_{kimh}^j - \sum_{t \in B3kih} V_{kiht}^j \text{---}, icI_k, k \in K_j, h = 1, \dots, 7, \\
 & \text{---} r_k Y_{kih}^j = \sum_{l \neq h, (h+1)} Z_{kihl}^j \text{---}, icI_k, k \in K_j, h = 1, \dots, 7, \\
 & \text{---} Z_{kihm}^j \leq Y_{kih}^j \text{---}, icI_k, k \in K_j, h = 1, \dots, 7, \\
 & \text{---} m \neq h, (h+1) \text{---} \\
 & \text{---} X_{ki}^j = \sum_h Y_{kih}^j \text{---}, icI_k, k \in K_j.
 \end{aligned}$$

Claim 12 (withdrawn):

~~The method of claim 9, in which the break and daily shift scheduling requirements are formulated into a set of constraints requiring that sufficient number of daily breaks and shifts are scheduled for agents from each agent skill group assigned to tours not requiring consecutive days off and consistent daily shift start times as:~~

$$QX_{knh}^j = \sum_{t \in QB1knh} QU_{knh}^j \text{---}, neF_k, icQI_k, k \in QK_j, h = 1, \dots, 7, \text{---}$$

$$\begin{aligned} QX_{knh}^j &= \sum_{t \in QB2knh} QW_{knh}^j, \quad n \in F_k, i \in QI_k, k \in QK_j, h = 1, \dots, 7, \\ QX_{knh}^j &= \sum_{t \in QB3knh} QV_{knh}^j, \quad n \in F_k, i \in QI_k, k \in QK_j, h = 1, \dots, 7, \\ \sum_{i \in QI_k} A_{kdh} Q_{ki}^j &= \sum_{n \in F_k} \sum_{i \in QI_k} QX_{knh}^j, \quad k \in QK_j, h = 1, \dots, 7. \end{aligned}$$

Claim 13 (withdrawn):

The method of claim 9, in which daily breaks, shift and days off scheduling requirements for tours requiring consistent daily shift start times are formulated into a set of constraints by defining pseudo-tours k in QK_j , each with one daily start time in QI_k and shift lengths in F_k as:

$$\begin{aligned} QX_{knh}^j &= \sum_{t \in QB1knh} QU_{knh}^j, \quad n \in F_k, i \in QI_k, k \in QK_j, h = 1, \dots, 7, \\ QX_{knh}^j &= \sum_{t \in QB2knh} QW_{knh}^j, \quad n \in F_k, i \in QI_k, k \in QK_j, h = 1, \dots, 7, \\ QX_{knh}^j &= \sum_{t \in QB3knh} QV_{knh}^j, \quad n \in F_k, i \in QI_k, k \in QK_j, h = 1, \dots, 7, \\ \sum_{i \in QI_k} A_{kdh} Q_{ki}^j &= \sum_{n \in F_k} \sum_{i \in QI_k} QX_{knh}^j, \quad k \in QK_j, h = 1, \dots, 7. \end{aligned}$$

Claim 14 (withdrawn):

The method of claim 9, in which the limited agent availability for agent group j and tour k , $D_k^{j, \max}$ and $QD_k^{j, \max}$, is formulated into a set of constraints as:

$$\begin{aligned} \sum_{i \in I_k} X_{ki}^j &\leq D_k^{j, \max}, \quad j \in J, k \in K_j, \\ \sum_{i \in QI_k} Q_{ki}^j &\leq QD_k^{j, \max}, \quad j \in J, k \in QK_j. \end{aligned}$$

Claims 15 (withdrawn):

~~The method of claim 9, in which fixed days off or early closure days for a tour is assured by setting values of associated days off variables to zero.~~

Claim 16 (withdrawn):

~~The method of claim 9, in which the objective function can be formulated to minimize cost, total agent time scheduled, total agent time scheduled weighted by agent skill group, total paid agent time scheduled, total paid agent time scheduled weighted by agent skill group, or to maximize agent preferences.~~

Claim 17 (withdrawn):

~~The method of claim 9, in which the objective function, and agent requirement constraint for each period and contact types are formulated to include an overage and an underage variable, and a penalty for each period and contact type with agent shortage in the objective function when there aren't enough number of agents to meet requirements by all contact types in all periods to be scheduled.~~

Claim 18 (withdrawn):

~~The method of claim 17, in which the total overage or underage for a plurality of contact types is limited to a pre-specified level.~~

Claim 19 (withdrawn):

~~A rounding algorithm (RA) to search for integer feasible solutions using a solution violating only some integrality requirements on some decision variables to a Mixed Integer Linear Programming (MILP) model of a workforce scheduling environment comprising the step of:~~

- ~~a) Obtaining the values of the decision variables in the solution found to the MILP model;~~

- ~~b) Rounding the fractional values of decision variables U, W, V, QX, QU, QW, QV, AND G down, and weekly tour variables X, and work pattern variables down if their fractional part is less than or equal to 0.50, and up if greater than 0.50, provided the agent availability constraints are satisfied;~~
- ~~c) Scheduling additional days off by increasing the values of Y and Z if additional days off needed to satisfy tour scheduling requirements, and additional daily shifts to have a shift scheduled for every agent who will be working on a given day based of the work pattern scheduled;~~
- ~~d) Scheduling additional daily breaks if the number of breaks of each type is not sufficient to satisfy break scheduling rules for a tour, and unscheduling breaks if more breaks are scheduled due to rounding;~~
- ~~e) Computing agent shortages and excesses for each contact type and planning period;~~
- ~~f) Computing agent availability for all agent groups, comparing them with the agent allocations to different contact types by the rounded values of allocation variables G, and allocating excess agents not accounted by allocation variables G to contact types;~~
- ~~g) Checking the solution constructed in steps (a) through (f) to find out if all agent requirements are met in every period and, when all requirements are met, eliminating all redundant agent tours and stopping with the integer feasible solution found;~~
- ~~h) Stopping with an infeasible solution if all available agents are scheduled;~~
- ~~i) Continuing to step (i) if there are agents available to schedule and there shortages in some periods for some contact types;~~
- ~~j) Finding all periods in which some contact types have shortages, finding an agent together with a complete tour schedule with work and non work days, daily shift start times and shift lengths, daily break times to reduce the agent shortages, and adding to the solution by updating the values of the decision variables to include the newly found tour schedule;~~
- ~~k) Repeating steps (h), (i), and (j) until an integer feasible solution satisfying all requirements if found;~~

- 1) ~~Examining the tours scheduled in the integer feasible solution found and eliminating redundant tours~~

Claim 20 (Withdrawn):

~~A computer program in a computer readable medium to be run on a computer for developing optimal schedules for a plurality of agents with a combination of defined skills, and work groups with part time/full time work requirements, a plurality of contact types requiring defined agent skills, a plurality of tour and shift scheduling rules, and a plurality of contact centers, comprising of:~~

- ~~means for acquiring agent and skill requirements for each period to be scheduled;~~
- ~~means for acquiring, for each contact center to be scheduled, operating hours, agent skill groups, agent work groups specifying part time/full time work patters, and scheduling paradigm for contact centers to specify whether each center is to be scheduled independently or a plurality of contact centers as a virtual contact center with combined;~~
- ~~means for acquiring tour and shift parameters and scheduling rules, and their availability to agent skill and work groups at different contact centers;~~
- ~~means for acquiring the objective criterion to be optimized and objective function parameters to quantify cost or benefit;~~
- ~~means for generating an MILP model by defining decision variables, parameters and sets, and then formulating the objective function, agent and skill requirement constraints, breaks constraints, days off and work pattern constraints, agent availability constraints, other schedule related constraints, and non-negativity and integrality constraints;~~
- ~~means for solving the MILP model using an optimization algorithm;~~
- ~~means for generating a detailed tour schedule using a terminal solution to the MILP model found by assigning daily shifts to work patterns scheduled, days off to weekly tours, and breaks to daily shifts;~~
- ~~means for assigning agents to specific tours.~~